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Transient motion of a dipole in a rotating flow

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The question of whether or not waves exist upstream of an obstacle that moves uniformly through an unbounded, incompressible, inviscid, unseparated, rotating flow is addressed by considering the development of the disturbed flow induced by a weak, moving dipole that is introduced into an axisymmetric, rotating flow that is initially undisturbed. Starting from the linearized equations of motion, it is shown that the flow tends asymptotically to the steady flow determined on the hypothesis of no upstream waves and that the transient at a fixed point is $O(1/t)$. It also is shown that the axial velocity upstream ($x < 0$) of the dipole as $x \to -\infty$ with t fixed is $O(|x|^{-3})$, as in potential flow, but is $O(|x|^{-1})$ as $t \to \infty$ with $|x|$ fixed. The results extend directly to closed obstacles of sufficiently small transverse dimensions and suggest the existence of a finite, parametric domain of no upstream waves for smooth, slender obstacles. The axial velocity in front of a small, moving sphere at a given instant in the transient régime is calculated and compared with Pritchard's laboratory measurements. The agreement is within the experimental scatter for Rossby numbers greater than about 0·3 even though the equivalence between sphere and dipole is exact only for infinite Rossby number.

1. Introduction

Does the uniform translation of an obstacle in an otherwise unbounded, incompressible, inviscid, rotating or stratified fluid induce upstream waves—in particular, *waves that produce a finite change in the conditions at an infinite distance upstream of the obstacle*? This question has been posed many times, but its answer remains controversial. It may be posited that no such waves exist in the limit of potential flow (no rotation or stratification); it also is generally accepted that such disturbances do exist for sufficiently strong rotation or stratification and are responsible for *blocked flows*, which are characterized by Taylor columns and upstream stagnation zones, respectively. The appropriate similarity parameter for a given body is either

$$k_0 = Nl/U, \tag{1.1a}$$

or

$$k_1 = 2\Omega l/U \equiv k, \tag{1.1b}$$

where l is a characteristic length, U is the translational speed, N is the intrinsic (Väisälä) frequency and is assumed constant, and Ω is the rotational speed; potential flow corresponds to $k_j \to 0$, blocked flow to $k_j \to \infty$. Both k_0 and k_1 are *reduced frequencies* or *wave numbers*; alternatively, $k_{0(1)}$ is an inverse Froude (Rossby) number. We replace k_1 by k in § 2 *et seq.*, wherein only rotating flow is considered.

It appears to be widely accepted by meteorologists [see Queney *et al.* (1960) or Yih (1965) for discussion and references] that an obstacle of height l does not produce upstream waves in a stratified flow if k_0 is sufficiently small. This proposition is supported both by the solution of relevant initial value problems (see Queney *et al.* 1960) and by experiment (Long 1955). It also appears to be generally accepted that upstream waves do arise for sufficiently large values of k_0. The latter proposition is supported by Bretherton's (1967) solution of the initial value problem for a circular cylinder as $k_0 \to \infty$ and by Long's (1955) experiments. The hypothesis of no upstream waves for an obstacle moving along the axis of a rotating flow, originally proposed and investigated by Long (1953), appears to be controversial for any $k_1 > 0$ [see Greenspan (1968), Miles (1969), and Pritchard (1969) for recent discussions].

Trustrum (1964) considers the transient development of rotating and stratified flows on the hypothesis of small disturbances (as in § 2 below) and gives solutions for stratified flow, in a channel of finite height, induced by either a line-sink or a line-source distribution in a plane normal to the flow and for rotating flow past a porous disk. She concludes that "the assumption of a uniform undisturbed upstream flow, which has been basic to most theories in both stratified flow and rotating fluids is probably not valid". She also notes, however, that "the solution for a [plane] dipole with its axis along the direction of the uniform stream... has no terms independent of x and so its influence does not extend to upstream infinity".

We regard the implications of this last statement as decisive, at least for bodies of sufficiently small transverse dimensions, by virtue of the established fact that the motion associated with either two-dimensional, stratified flow over an obstacle of finite cross-section or rotating flow past an axisymmetric body of finite volume can be attributed to an equivalent distribution of axial dipoles (Miles & Huppert 1969; Miles 1969). This holds also for flow past any closed stream surface, but not for separated flow with an infinite wake, for which the inviscid solution contains a source term. Motivated by this fact, we consider here the transient development of the disturbance following the abrupt introduction of an axial dipole into a previously uniform, rotating flow. We anticipate that the resulting, steady-state disturbance is essentially identical with that obtained by Fraenkel (1956) for steady flow on the hypothesis of no upstream waves. The corresponding development for a two-dimensional stratified flow follows by analogy.†

It would be sufficient for our primary purpose to infer the non-existence of cylindrical waves in the asymptotic solution for the dipole as $t \to \infty$ either from the non-existence of a component proportional to $\delta(\alpha)$ in the Fourier transform (α is the wave-number) of that solution or from the temporal invariance of the dipole moment of the developing solution (which we prove in § 4 below). It appears to be worthwhile to go a bit further, however, and to discuss (in § 4) the form of

† Crapper (1959) demonstrates the absence of upstream waves or a dipole in a stratified flow but does not consider the transient development of the solution.

the developing solution, especially in the neighbourhood of the upstream axis, and to obtain (in § 5) an explicit, asymptotic approximation to the transient component of the solution.

We infer from the analysis of § 4 that the axial velocity upstream ($x < 0$) of a moving dipole as $x \to -\infty$ with t fixed is $O(|x|^{-3})$, as in potential flow, but is $O(|x|^{-1})$ as $t \to \infty$ with $|x|$ fixed and is typically much larger than in potential flow. We give an explicit example in § 6, where we calculate the axial velocity in front of a small, moving sphere at a given instant and compare the results with the laboratory observations of Pritchard (1969). The agreement is within the experimental scatter over the complete range of the data, $0{\cdot}8 < k < 3{\cdot}2$, which is perhaps better than might have been expected for an approximation (of the sphere by a dipole) that is strictly valid only for $k \to 0$. We emphasize that the comparison between theory and experiment is for an accelerated flow, in which separation may reasonably be expected to be less important than in the corresponding steady flow for a sphere. It does suggest, nevertheless, that the axial flow observed upstream of a moving body in a rotating flow of finite dimensions might easily be interpreted as a Taylor column under circumstances in which it could be satisfactorily explained without discarding the hypothesis of no upstream waves.

We infer from the analysis of §§ 5 and 6 that the hypothesis of no upstream waves is valid for unseparated flow and sufficiently small disturbances. The latter restrictions appear to be reasonable for smooth, slender obstacles of axial length l and transverse diameter ϵl and sufficiently small values of both ϵ and $k_j \epsilon$ [*slender-body theory* applies in the limit $\epsilon \to 0$ with k_j fixed; see Miles & Huppert (1969) for $j = 0$ and Miles (1969) for $j = 1$]. The quantitative meaning of *sufficiently small* almost certainly must be determined by experiment, but, in any event, it seems likely that a steady flow calculated on the hypothesis of no upstream disturbance is unstable for values of $k_j \epsilon$ larger than that value of $k_j \epsilon$ at which the streamlines exhibit local reversals (see Miles & Huppert 1969 and Miles 1969).

The assumption of steady, unseparated flow is not realistic for bluff obstacles, such as a sphere (even though the accelerated flow past such obstacles may remain unseparated). Separation of the flow past such obstacles typically yields a semi-infinite, trailing wake (and perhaps also a forward wake), the inviscid representation of which requires a source (Stewartson 1968).

2. Initial value problem

We render all lengths and time dimensionless by reference to the characteristic length l and the characteristic time l/U, where U is the basic flow speed.† We satisfy the equation of continuity by deriving the x and y components of the perturbation velocity from a dimensionless stream function ψ according to

$$\{u - U, v\} = Uy^{-1}\{-\psi_y, \psi_x\}, \tag{2.1}$$

† The length l is arbitrary for the dipole problem, in consequence of which k could be eliminated from the dipole solution by choosing $l = (U/2\Omega)$. Such a choice would obscure the limit of potential flow ($k \to 0$).

where x is the axial co-ordinate, y is the cylindrical radius, and the subscripts imply partial differentiation.

Substituting (2.1) into Euler's equations, invoking the hypothesis of small disturbances, and eliminating the pressure, we obtain (cf. Trustrum 1964, after allowing for differences of notation)

$$L\psi \equiv [(\partial_t + \partial_x)^2 (\partial_x^2 + y\partial_y y^{-1}\partial_y) + k^2\partial_x^2]\,\psi = 0, \tag{2.2}$$

where the operators ∂_t, ∂_x and ∂_y imply partial differentiation, and k is given by (1.1).

The azimuthal (swirl) component of the velocity in the axisymmetric rotating flow is given by

$$w = \Omega l[y - 2y^{-1}\partial_x(\partial_t + \partial_x)^{-1}\psi] \tag{2.3a}$$

$$= \Omega l\left[y - 2y^{-1}\,\partial_x \int_0^t \psi(x-\tau, t-\tau)\,d\tau\right]. \tag{2.3b}$$

The boundary condition corresponding to the introduction, at $t = 0$, of a dipole of unit moment at $x = y = 0$ is (see below)

$$\psi = \delta(x)\,H(t) \quad (y = 0), \tag{2.4}$$

where $\delta(x)$ is Dirac's delta function, and $H(t)$ is Heaviside's step function. We also impose the finiteness conditions (which could be made somewhat weaker)

$$|\psi, \psi_x, \psi_y| \to 0 \quad (|x| \text{ or } y \to \infty). \tag{2.5}$$

Invoking the assumption that the flow is uniform for $t < 0$, we obtain the initial conditions

$$\psi = \partial_t\psi = 0 \quad (t = 0) \tag{2.6}$$

except at the singular point, $x = y = 0$.

Referring to (2.4), we observe that the flow is initially irrotational and that the axial-dipole solution in such a flow is given by

$$\psi = \tfrac{1}{2}r^{-1}\sin^2\theta, \tag{2.7}$$

where r and θ are spherical polar co-ordinates:

$$x = r\cos\theta, \quad y = r\sin\theta. \tag{2.8}$$

Letting $y \downarrow 0$ in (2.7) and observing that the dipole moment is unity,

$$\int_{-\infty}^{\infty} \psi_0(x, y)\,dx = 1, \tag{2.9}$$

we obtain

$$\lim_{y\downarrow 0} \psi_0(x, y) = \delta(x). \tag{2.10}$$

3. Integral transform solution

We define the Laplace and Fourier transform operators and their inverses according to

$$\mathscr{L}_t(\;) = \int_0^\infty e^{-\sigma t}(\;)\,dt \quad (\mathscr{R}\sigma > 0), \tag{3.1a}$$

$$\mathscr{L}_t^{-1}(\;) = (2\pi i)^{-1} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{\sigma t}(\;)\, d\sigma \quad (\gamma > 0), \tag{3.1b}$$

$$\mathscr{L}_x(\;) = \int_{-\infty}^{\infty} e^{-i\alpha x}(\;)\, dx, \quad \mathscr{L}_x^{-1}(\;) = (2\pi)^{-1} \int_{-\infty}^{\infty} e^{i\alpha x}(\;)\, d\alpha, \tag{3.2a,b}$$

and the transform of ψ according to

$$\Psi(y, \alpha, \sigma) = \mathscr{L}_t \mathscr{L}_x \psi(x, y, t), \tag{3.3}$$

where σ is a complex variable with a positive real part, γ is a real number that places the path of integration to the right of all singularities of Ψ in the σ plane,

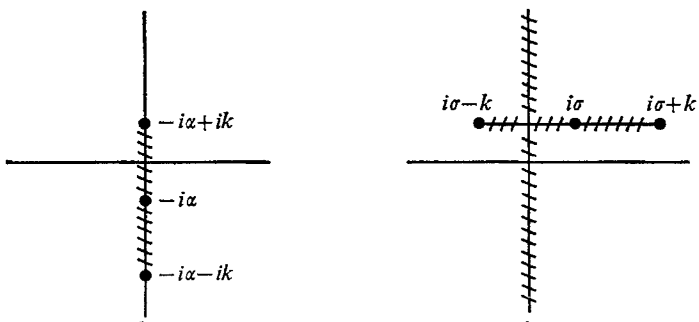

FIGURE 1. The branch points of λ in the σ plane for real α and in the α plane for $\mathscr{R}\sigma > 0$.

and α and β are real. The properties of these operators, as well as the transform pairs required in the subsequent development, are given in Erdélyi, Magnus, Oberhettinger & Tricomi (1953), to which we refer by the prefix EMOT, followed by the required entry number. We do not seek complete mathematical rigor, but all subsequent operations on singular functions can be justified through generalized-function theory (Lighthill 1959).

Transforming (2.2) and (2.4) and invoking (2.6), we obtain

$$(y\partial_y y^{-1}\partial_y - \lambda^2)\Psi = 0 \tag{3.4}$$

and

$$\Psi = \sigma^{-1} \quad (y = 0), \tag{3.5}$$

where

$$\lambda^2 = \alpha^2\{1 + k^2(\sigma + i\alpha)^{-2}\}. \tag{3.6}$$

The required solution of (3.4) and (3.5), subject to the finiteness condition (2.5) at $y = \infty$, is

$$\Psi = \sigma^{-1}\lambda y K_1(\lambda y) \equiv \sigma^{-1}E(\lambda y) \quad (\mathscr{R}\lambda > 0), \tag{3.7}$$

where K_1 is a modified Bessel function. The branch cuts of λ, *qua* function of either α or σ, are determined by the restriction $\mathscr{R}\lambda > 0$ ($\mathscr{R} \equiv$ *real part of*) and are sketched in figure 1.

4. Initial development

Invoking the initial value theorem for the Laplace transform and restricting α to real values, we obtain

$$\lim_{t \downarrow 0} \mathscr{L}_x \psi = \lim_{\sigma \to \infty} \sigma \mathscr{L}_t \mathscr{L}_x \psi = E(|\alpha|\, y). \tag{4.1}$$

Invoking EMOT 1.4(1) and 1.12(41), we obtain

$$\lim_{t\downarrow 0} \psi(x,y,t) = \psi_0(x,y), \tag{4.2}$$

where ψ_0 is given by (2.7).

We now introduce the auxiliary function

$$F(\eta,t) = \mathscr{L}_t^{-1}\{E[\eta(1+\sigma^{-2})^{\frac{1}{2}}]-E(\eta)\}, \tag{4.3}$$

where the real part of $(1+\sigma^{-2})^{\frac{1}{2}}$ is positive. Invoking (4.1), (4.2), and the scaling and shifting theorems for the Laplace transform, we obtain the inverse Laplace transform of (3.7) in the form

$$\mathscr{L}_x\{\psi-\psi_0\} = k\int_0^t e^{-i\alpha\tau}F(|\alpha|\,y,k\tau)\,d\tau. \tag{4.4}$$

Invoking (3.2 b) and resolving the α integrand into odd and even parts, we obtain

$$\psi(x,y,t) = \psi_0(x,y)+(k/\pi)\int_0^\infty d\alpha\int_0^t F(\alpha y,k\tau)\cos\{\alpha(x-\tau)\}\,d\tau. \tag{4.5}$$

We remark that $F(0,k\tau)\equiv 0$, by virtue of which ψ does not contain a cylindrical (x-independent) component (the existence of which would require $\mathscr{L}_x\psi$ to have a delta-function singularity at $\alpha=0$); moreover, the dipole moment of ψ is equal to that of ψ_0, since

$$\int_{-\infty}^\infty \{\psi-\psi_0\}\,dx = \mathscr{L}_x\{\psi-\psi_0\}_{\alpha=0}\equiv 0. \tag{4.6}$$

We may interpret $\psi-\psi_0$ as a superposition, over $\alpha=(0,\infty)$ and $\tau=(0,t)$, of individual waves of wave-number α and amplitude $kF(\alpha y,k\tau)$, each of which originates at $t=\tau$ and then moves downstream with speed U or, equivalently, remains at rest with respect to the fluid at infinity. We anticipate that the cumulative effect of these waves yields a system of standing waves in the lee of the dipole (or obstacle).

The result of (4.5) is directly useful for $kyt \ll |t-x|$. Substituting (4.3) into (4.5) and carrying out the α and σ integrations in that order [EMOT 1.12(41), 5.4(23)], we obtain

$$\psi = \psi_0+\tfrac{1}{2}ky\partial_y\int_0^t d\tau\,\mathscr{L}_{k\tau}^{-1}\{R^{-1}-\sigma(\sigma^2R^2+y^2)^{-\frac{1}{2}}\} \tag{4.7a}$$

$$= \psi_0+\tfrac{1}{2}ky\partial_y\int_0^t (y/R^2)\,J_1(ky\tau/R)\,d\tau, \tag{4.7b}$$

where
$$R(t) = \{(x-t)^2+y^2\}^{\frac{1}{2}}, \tag{4.8}$$

and $R=R(\tau)$ in (4.7 a,b). Letting $kyt/R\downarrow 0$ in (4.7 b), we obtain

$$\psi\to\psi_0-\tfrac{1}{4}k^2y^2\left\{\frac{1}{R(t)}+\frac{t(t-x)}{R^3(t)}-\frac{1}{R(0)}\right\}\quad (kyt/R\downarrow 0). \tag{4.9}$$

We return to this result in § 6 below.

5. Asymptotic development

The dominant contributions to ψ as $t \to \infty$ are associated with the singularities of $\sigma^{-1}E(\lambda y)$ at $\sigma = 0$, $-i\alpha$ and $-i\alpha \pm ik$. We separate out the contribution from $\sigma = 0$, say ψ_∞, as follows:

$$\psi_\infty = \mathscr{L}_x^{-1}\mathscr{L}_t^{-1}\{\sigma^{-1}E[y(\alpha^2-k^2)^{\frac{1}{2}}]\} \tag{5.1a}$$

$$= \pi^{-1}\mathscr{R}\int_0^\infty E[y(\alpha^2-k^2)^{\frac{1}{2}}]\, e^{i\alpha x}\, d\alpha, \tag{5.1b}$$

where (we now relax the original restriction to real α) the path of integration in the α plane is indented under the branch points at $\alpha = \pm k$ (the branch points of λ at $\alpha = i\sigma \pm k$ tend to the real axis from above as $\sigma \to 0$ in $\mathscr{R}\sigma > 0$). Having accounted for the singularity at $\sigma = 0$, we modify the representation of (4.5) to obtain

$$\psi = \psi_\infty - (k/\pi)\int_0^\infty d\alpha \int_t^\infty F(\alpha y, k\tau)\cos\{\alpha(x-\tau)\}\, d\alpha, \tag{5.2}$$

where
$$F(\eta, t) = \mathscr{L}_t^{-1}\{E[\eta(1+\sigma^{-2})^{\frac{1}{2}}]\} \quad (t > 0). \tag{5.3}$$

The function F determined by (5.3) is identical with F in § 4 for $t > 0$, and the dominant terms in its asymptotic approximation are associated with the singularity at $\sigma = 0$ (corresponding to the singularity at $\sigma = i\alpha$ in the original σ plane).

Expanding E about $\sigma = 0$ and invoking EMOT 5.5(37), we obtain

$$F(\eta, t) \sim \mathscr{L}_t^{-1}\{(\pi\eta/2\sigma)^{\frac{1}{2}}\exp(-\eta/\sigma)\} = (\eta/2t)^{\frac{1}{2}}\cos\{2(\eta t)^{\frac{1}{2}}\}. \tag{5.4}$$

[We note that the contributions of the singularities of $\sigma = \pm i$ ($\sigma = -i\alpha \pm ik$ in the original σ plane) to F are $O(\eta^2 t^{-2})$.] Substituting (5.4) into (5.2), approximating the integral with respect to τ by integration by parts, and invoking EMOT 2.7(18)† for the α integration, we obtain

$$\psi \sim \psi_\infty + \left\{\frac{ky}{2\pi t(t-x)}\right\}^{\frac{1}{2}}\cos\left(\frac{kyt}{t-x}+\tfrac{1}{4}\pi\right) \quad (t, t-x \to \infty) \tag{5.5a}$$

$$\sim \psi_\infty + (ky/2\pi)^{\frac{1}{2}}\, t^{-1}\cos(ky+\tfrac{1}{4}\pi) \quad (t \to \infty, \quad x \ll t). \tag{5.5b}$$

We remark that (5.5a, b) are not uniformly valid for $ky \to \infty$; however, this limit is of little interest in the present context.

We conclude from (5.5) that ψ tends asymptotically to the steady-state, dipole solution ψ_∞ as $t \to \infty$. The dipole solution ψ_∞ is due originally to Fraenkel (1956). It has the asymptotic representations

$$\psi_\infty \sim kH(x)\sin kr \sin^2\theta \quad (kr \to \infty), \tag{5.6}$$

and
$$\psi_\infty \sim -\tfrac{1}{2}kx^{-1}yJ_1(ky) \quad (x \to -\infty), \tag{5.7}$$

provided that θ is bounded away from $\frac{1}{2}\pi$ in (5.6) and y is bounded in (5.7). We also note that

$$\psi_\infty \to \psi_0 \quad (kr \to 0). \tag{5.8}$$

† This entry contains a misprint and should read $(\pi/y)^{\frac{1}{2}}\cos(\frac{1}{4}a^2y^{-1}+\frac{1}{4}\pi)$.

6. Comparison with experiment

The dipole solution ψ provides the basis of the solution of (2.2) and (2.4) for an axisymmetric body of scale length l as $k \to 0$ by virtue of the facts that: ψ tends to a potential function, say ψ_p, as $kr \to 0$; ψ_p for any closed body exhibits a dipole behaviour as $r \to \infty$, say

$$\psi_p(x,y) \sim D\psi_0(x,y) \quad (r \to \infty), \tag{6.1}$$

where D is the *dipole moment* of the body with respect to axial translation. It follows that

$$\psi(x,y,t) \sim D\psi_D(x,y,t) \quad (k \to 0) \tag{6.2a}$$

$$\sim D\psi_0(x,y) \quad (kr \to 0), \tag{6.2b}$$

where ψ_D is the dipole solution of §§ 3–6 above, is an outer, asymptotic approximation for a small, closed obstacle of dipole moment $DH(t)$ that may be matched to the inner approximation $\psi_p(x,y)\,H(t)$ in $1 \ll r \ll 1/k$.

Pritchard (1969, figure 6) reports measurements of the axial velocity upstream of a moving sphere in a circular tube of large radius (9.6 sphere radii). He compares his results with a theoretical calculation (Greenspan 1968, § 4.3) for a disk moving at low speeds ($k \to \infty$) on the "assumption that, at high Rossby numbers, the waves whose group velocity is less than the velocity of the obstacle are eliminated from the Taylor column ahead of the obstacle". This assumption implies that the upper limit in Greenspan's (4.3.13) may be replaced by k (in the present notation), in consequence of which the axial velocity u/U is rendered independent of both x and t and depends only on k. This contrasts with Greenspan's original result, which implies an asymptotic representation of u/U as $k \to \infty$, $x \to \infty$ and $t \to \infty$ and is independent of U, and with Pritchard's own measurements, which imply a significant dependence of u/U in both x and t. Despite (what appears to be) the essentially *ad hoc* character of Pritchard's interpretation of Greenspan's theoretical result, the general trends of this interpretation and the observations are similar over the entire range of comparison ($0{\cdot}8 < k < 3{\cdot}2$), and the agreement is within the experimental scatter for $0{\cdot}8 < k < 1{\cdot}3$.

We compare Pritchard's measurements with the approximation provided by (6.2) above, in which l is the radius of the sphere and $D = 1$ (the latter equivalence is strictly correct only for $k \to 0$). Substituting (2.7a) and (4.9) into (2.1) and letting $y \to 0$, we obtain

$$(U-u)/U \to |x|^{-3} + \tfrac{1}{2}k^2\{|x|^{-1} - |t-x|^{-1} - t(t-x)\,|t-x|^{-3}\}$$

$$(kyt\,|t-x|^{-1} \to 0) \tag{6.3a}$$

$$= |x|^{-3} + \tfrac{1}{2}k^2|x|^{-1}t^2(t-x)^{-2} \quad (x < 0, \quad y = 0). \tag{6.3b}$$

This result is plotted in figure 2 as a function of the Rossby number, $1/k$, for $|x| = 4$ and $kt = 8\pi$ (true time $= 4\pi/\Omega$) and compared with Pritchard's experimental points. The agreement is within the experimental scatter over the entire range of comparison, $0{\cdot}8 < k < 3{\cdot}2$, despite the fact that the equivalence between dipole and sphere is theoretically correct for only $k \to 0$. The agreement for the larger values of k, especially $k > 2$, may be fortuitous, as *steady-flow* calculations of the dipole moment (Stewartson 1958; Miles 1969) imply that it is a decreasing function of k. We again emphasize that the implicit assumption of unseparated flow may reasonably be expected to hold in accelerated, but not in steady, flow.

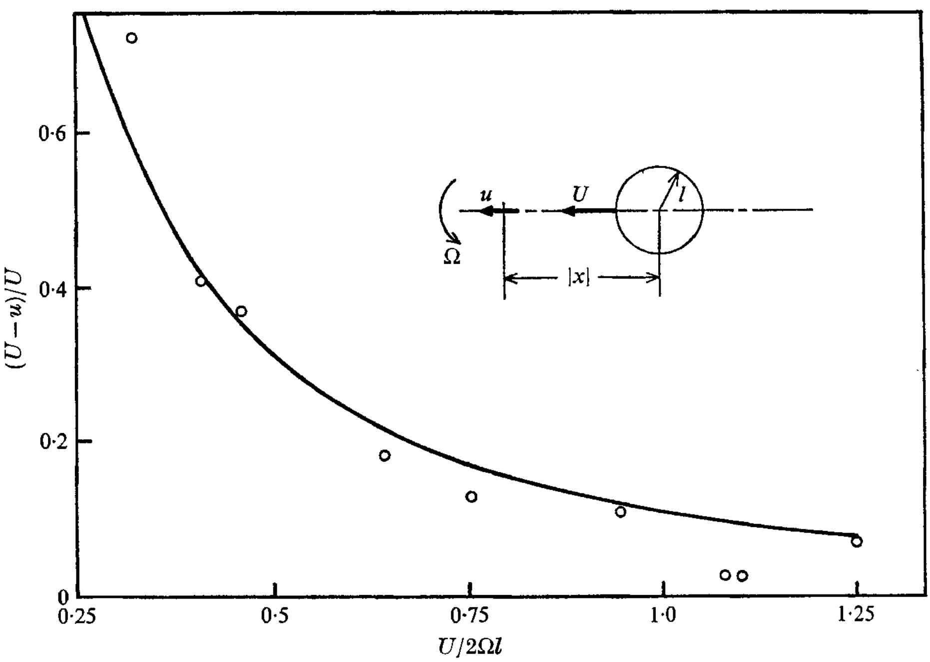

FIGURE 2. The axial velocity ahead of a sphere for $|x| = 4$ and $kt = 8\pi$, as calculated from (6.3*b*) on the hypothesis that the sphere can be approximated by a single dipole (solid curve) and as observed by Pritchard (1969) (small circles).

Letting $t/|x| \to \infty$ in (5.1*a*), we obtain the steady-state velocity (see Miles 1969 for details)

$$(U-u)/U \sim |x|^{-3} + \tfrac{1}{2}k^2|x|^{-1} \quad (t/|x| \to \infty, \quad y = 0). \tag{6.4}$$

The term $\frac{1}{2}k^2|x|^{-1}$, which represents the effect of rotation, is larger than $|x|^{-3}$, which would be the only component for an irrotational flow, for $|x| > \sqrt{2}/k$; moreover, the contrast is even larger for steady flow than during the transient development. This reinforces the suggestion made in § 1, namely that the axial flow observed upstream of a moving body in a rotating flow of finite dimensions might easily be interpreted as a Taylor column under circumstances in which it could be satisfactorily explained without discarding the hypothesis of no upstream waves.

The dominant term in (6.4) has a rather simple generalization for points off the axis, which may be inferred from (5.7), namely

$$(U-u)/U \sim \tfrac{1}{2}k^2|x|^{-1}J_0(ky) \quad (t/|x| \to \infty), \tag{6.5}$$

which predicts a (first) reversal of the induced flow at $ky = 2{\cdot}40$. Pritchard (1969, figure 4) reports measurements of the radial velocity profile for $k = 2{\cdot}41$ that exhibit such a reversal at $y \doteqdot 1{\cdot}1$. The zero-Rossby-number, disk model, on the other hand, predicts a first reversal at $y = 1\cdot7\text{–}1\cdot8$ ($ky \doteqdot 4\cdot2$). The agreement provided by (6.5) could be coincidental; moreover, the measured velocity profile does not represent a steady state (this discrepancy is more important at larger distances from the sphere and may be responsible for the less satisfactory agreement between the observed and theoretical profiles for $ky > 2\cdot4$). Additional data, especially for smaller values of k, could resolve this point.

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REFERENCES

Bretherton, F. P. 1967 The time-dependent motion due to a cylinder moving in an unbounded rotating or stratified fluid. *J. Fluid Mech.* **28**, 545–70.

Crapper, G. D. 1959 A three-dimensional solution for waves in the lee of mountains. *J. Fluid Mech.* **6**, 51–76.

Erdélyi, A., Magnus, W., Oberhettinger, F. & Tricomi, F. G. 1953 *Tables of Integral Transforms*, vols. 1 and 2. New York: McGraw-Hill.

Fraenkel, L. E. 1956 On the flow of rotating fluid past bodies in a pipe. *Proc. Roy. Soc.* A **233**, 506–26.

Greenspan, H. 1968 *The Theory of Rotating Fluids*. Cambridge University Press.

Lighthill, M. J. 1959 *Introduction to Fourier Analysis and Generalized Functions*. Cambridge University Press.

Long, R. R. 1953 Steady motion around a symmetrical obstacle moving along the axis of a rotating fluid. *J. Meteor.* **10**, 197–203.

Long, R. R. 1955 Some aspects of the flow of stratified fluids. III. *Tellus*, **7**, 341–57.

Miles, J. W. 1969 The lee-wave régime for a slender body in a rotating flow. *J. Fluid Mech.* **36**, 265–88.

Miles, J. W. & Huppert, H. E. 1969 Lee waves in a stratified flow. Part 4. *J. Fluid Mech.* **35**, 497–526.

Pritchard, W. 1969 The motion generated by an obstacle moving along the axis of a uniformly rotating fluid. *J. Fluid Mech.* **39**, 443.

Queney, P., Corby, G. A., Gerbier, N., Koschmieder, H. & Zierep, J. 1960 *The Airflow over Mountains*. Geneva: World Meteorological Organization.

Stewartson, K. 1958 On the motion of a sphere along the axis of a rotating fluid. *Quart. J. Mech. and Appl. Math.* **11**, 39–51.

Stewartson, K. 1968 On inviscid flow of a rotating fluid past an axially-symmetric body using Oséen's equations. *Quart. J. Mech. and Appl. Math.* **21**, 353–73.

Trustrum, K. 1964 Rotating and stratified fluid flow. *J. Fluid Mech.* **19**, 415–32.

Yih, C.-S. 1965 *Dynamics of Nonhomogeneous Fluids*. New York: Macmillan.